

Phase Noise Measurements and its Limitations

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Introduction

One of the important parameters for oscillators is phase noise as it is critical to most applications.¹⁻²² There are several methods for measuring phase noise which will be covered here including the pros and cons of each technique. Example measurements on various types of systems will also be compared including their accuracy and speed.

The pioneer in Phase Noise measurement was Hewlett Packard^{1, 2}. Modern Phase Noise systems use the correlation principle but this method has some drawbacks. The most direct and most sensitive method to measure the spectral density of phase noise, $S_{\Delta\theta}(f_m)$, requires two sources – one or both of them may be the device(s) under test – and a double balanced mixer used as a phase detector (see Figure 1). The RF and LO input to the mixer should be in phase quadrature, indicated by 0 Vdc at the IF port. Good quadrature assures maximum phase sensitivity K_θ and minimum AM sensitivity. With a linear mixer, K_θ equals the peak voltage of the sinusoidal beat signal produced when both sources are frequency offset.

When both signals are set in quadrature, the voltage ΔV at the IF port is proportional to the fluctuating phase difference between the two signals.

$$\Delta\theta_{rms} = \frac{1}{K_\theta V_{rms}} \quad (1)$$

$$S_{\Delta\theta}(f_m) = \frac{(\Delta V_{rms})^2(1Hz)}{V_{B\ peak}^2} \frac{1}{2} \frac{(\Delta V_{rms})^2(1Hz)}{V_{B\ rms}^2} \quad (2)$$

$$L(f_m) = \frac{1}{2} S_{\Delta\theta}(f_m) = \frac{1}{4} \frac{(\Delta V_{rms})^2(1Hz)}{V_{B\ rms}^2} \quad (3)$$

where K_θ = phase detector constant,

$$= V_{B\ peak} \text{ for sinusoidal beat signal}$$

The calibration of the wave analyzer or spectrum analyzer can be read from the equations above. For a plot of $L(f_m)$ the 0dB reference level is to be set 6dB above the level of the beat signal. The -6dB offset has to be corrected by +1.0dB for a wave analyzer and by +2.5dB for a spectrum analyzer with log amplifier and average detector. In addition, noise bandwidth corrections may have to be applied.

Since the phase noise of both sources is measured in this system, the phase noise performance of one of them needs to be known for definite data on the other source. Frequently, it is sufficient to know that the actual phase noise of the dominant source cannot deviate from the measured data by more than 3 dB. If three unknown sources are available, three measurements with three different source combinations yield sufficient data to calculate accurately each individual performance.

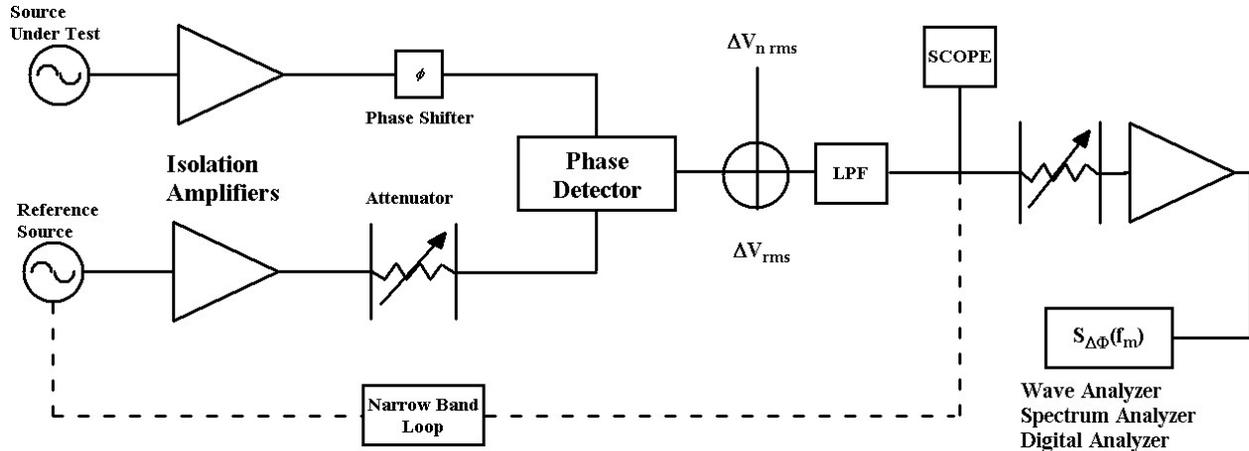


Figure 1: Phase Noise system with two sources maintaining phase quadrature

Figure 1 indicates a narrowband phase-locked loop that maintains phase quadrature for sources that are not sufficiently phase stable over the period of the measurement. The two isolation amplifiers should prevent injection locking of the sources. Residual phase noise measurements test one or two devices, such as amplifiers, dividers or synthesizers, driven by one common source. Since this source is not free of phase noise, it is important to know the degree of cancellation as a function of Fourier frequency.

The noise floor of the system is established by the equivalent noise voltage ΔV_n at the mixer output. It represents mixer noise as well as the equivalent noise voltage of the following amplifier:

$$L_{system}(f_m) = \frac{1}{4} \frac{(\Delta V_{n,rms})^2 (1Hz)}{V_{B,rms}^2} \quad (4)$$

Noise floors close to -180dBc can be achieved with a high-level mixer and a low-noise port amplifier. The noise floor increases with f_m^{-1} due to the flicker characteristic of ΔV_n . System noise floors of -166dBc at 1 kHz have been realized. This is the limitation of the previous approach.

In measuring low-phase-noise sources, a number of potential problems have to be understood to avoid erroneous data:

- If two sources are phase locked to maintain phase quadrature, it has to be ensured that the lock bandwidth is significantly lower than the lowest Fourier frequency of interest.
- Even with no apparent phase feedback, two sources can be phase locking (injection locked), resulting in suppressed close-in phase noise.
- AM noise of the RF signal can come through if the quadrature setting is not maintained sufficiently.
- Deviation from the quadrature setting will also lower the effective phase detector constant.
- Nonlinear operation of the mixer results in a calibration error and will add noise.
- A non-sinusoidal RF signal causes K_θ to deviate from $V_{B,peak}$
- The amplifier or spectrum analyzer input can be saturated during calibration or by high spurious signals such as line frequency multiples.

- Closely spaced spurious signals such as multiples of 60 Hz may give the appearance of continuous phase noise when insufficient resolution and averaging are used on the spectrum analyzer.
- Impedance interfaces should remain unchanged when going from calibration to measurement.
- In residual measurement system phase, the noise of the common source might be insufficiently canceled due to improperly high delay-time differences between the two branches.
- Noise from power supplies for devices under test or the narrowband phase-locked loop can be a dominant contributor of phase noise.
- Peripheral instrumentation such as the oscilloscope, analyzer, counter, or DVM can inject noise.
- Microphonic noise might excite significant phase noise in devices.

Despite all these hazards, automatic test systems have been developed and operated successfully for years.⁴ **Figure 2** shows a system that automatically measures the residual phase noise of the 8662A synthesizer. It is a residual test, since both instruments use one common 10MHz referenced oscillator. Quadrature setting is conveniently controlled by probing the beat signal with a digital voltmeter and stopping the phase advance of one synthesizer when the beat signal voltage is sufficiently close to zero.

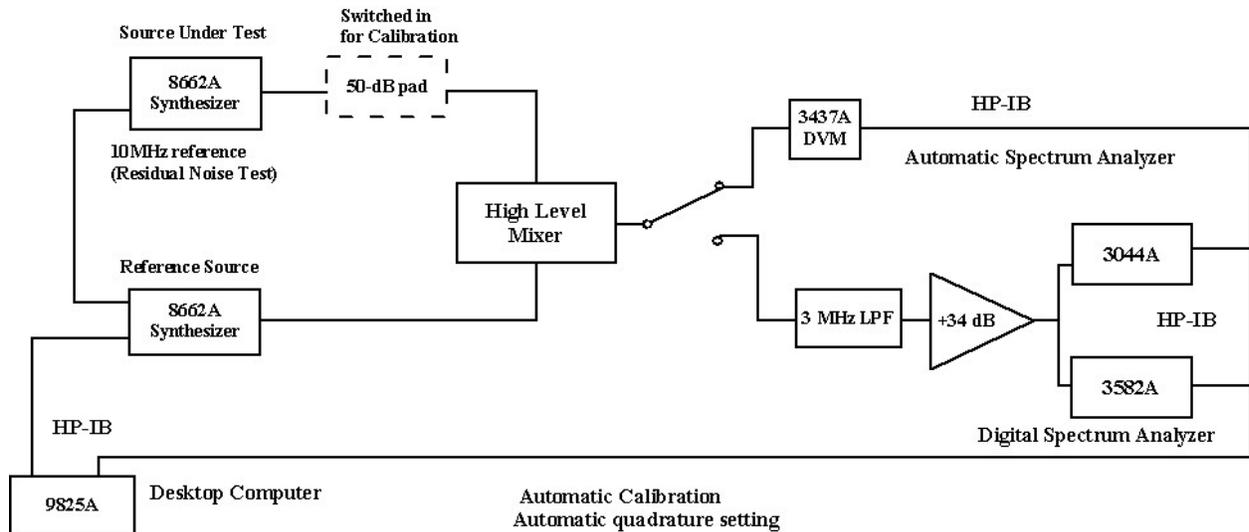


Figure 2: Automatic system to measure residual phase noise of two 8662A synthesizers (Courtesy of Hewlett-Packard Company)

Doing Measurements

As shown in **Figure 3**, the Hewlett Packard test equipment HP3047A was built around a double-balanced mixer which can handle +23dBm of power.

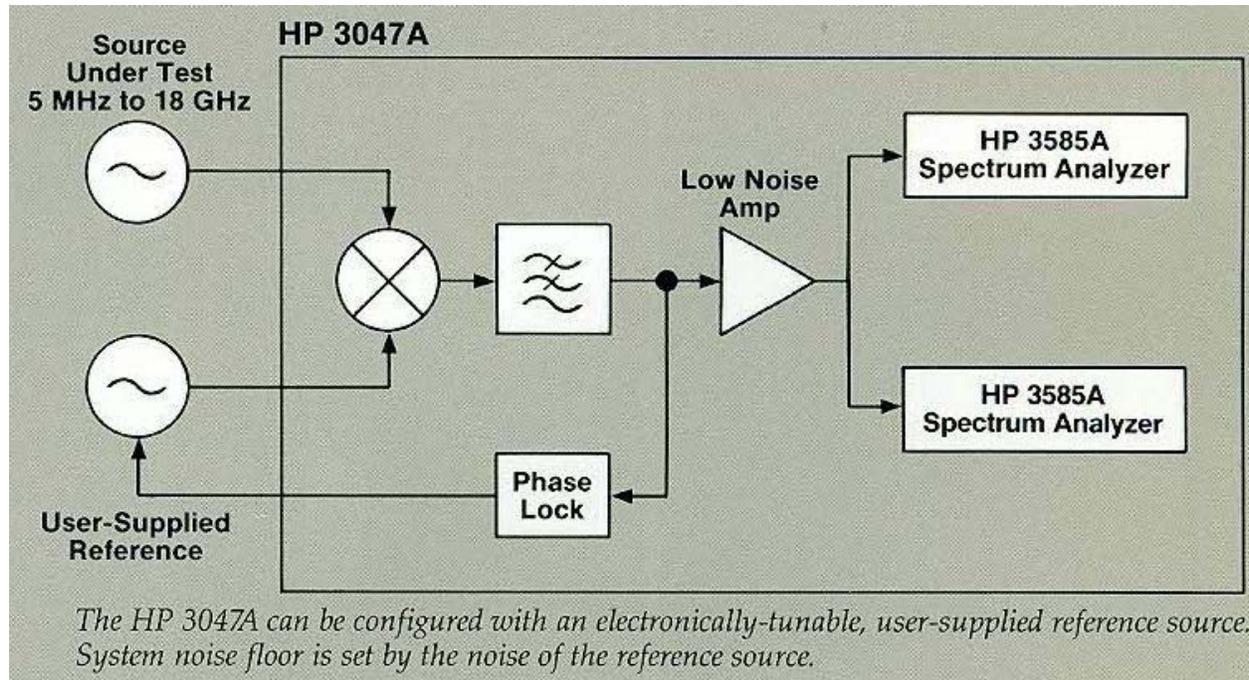


Figure 3: Block Diagram of HP3047A

While this HP equipment is no longer manufactured and has been replaced by the Agilent E5052A and E5052B, this system shows some interesting features.

1. The noise floor is about -177dBc/Hz and the resolution, shown in **Figure 4** is approximately -180 dBc/Hz
2. The limit of the dynamic range is $(P_{out} \text{ (dBm)} + kT - NF)$, (where $NF = NF_{osc} + NF_{amp}$), which in the case of a 0 dBm oscillator would be -174 dBc or relative to one sideband -177 dBc - NF. Assuming a signal to be +20 dBm, you then have the dynamic range of approximately 200 dBc - NF.
3. The term NF_{osc} refers to the large signal noise figure of the oscillator, which can vary from 2dB at 10 MHz to 6 dB at 100 MHz or even higher at 1000 MHz. As an example the far out SSB noise at 1000 MHz is typically, -172dBc/Hz at 5dBm of output power.
4. The mixer and the post amplifier can easily get into compression which raises the noise floor. In the previous publications for this system, this effect was not considered. It was found that some of the measurements done with this system at low frequencies were off, the crystal oscillators in question were actually better.

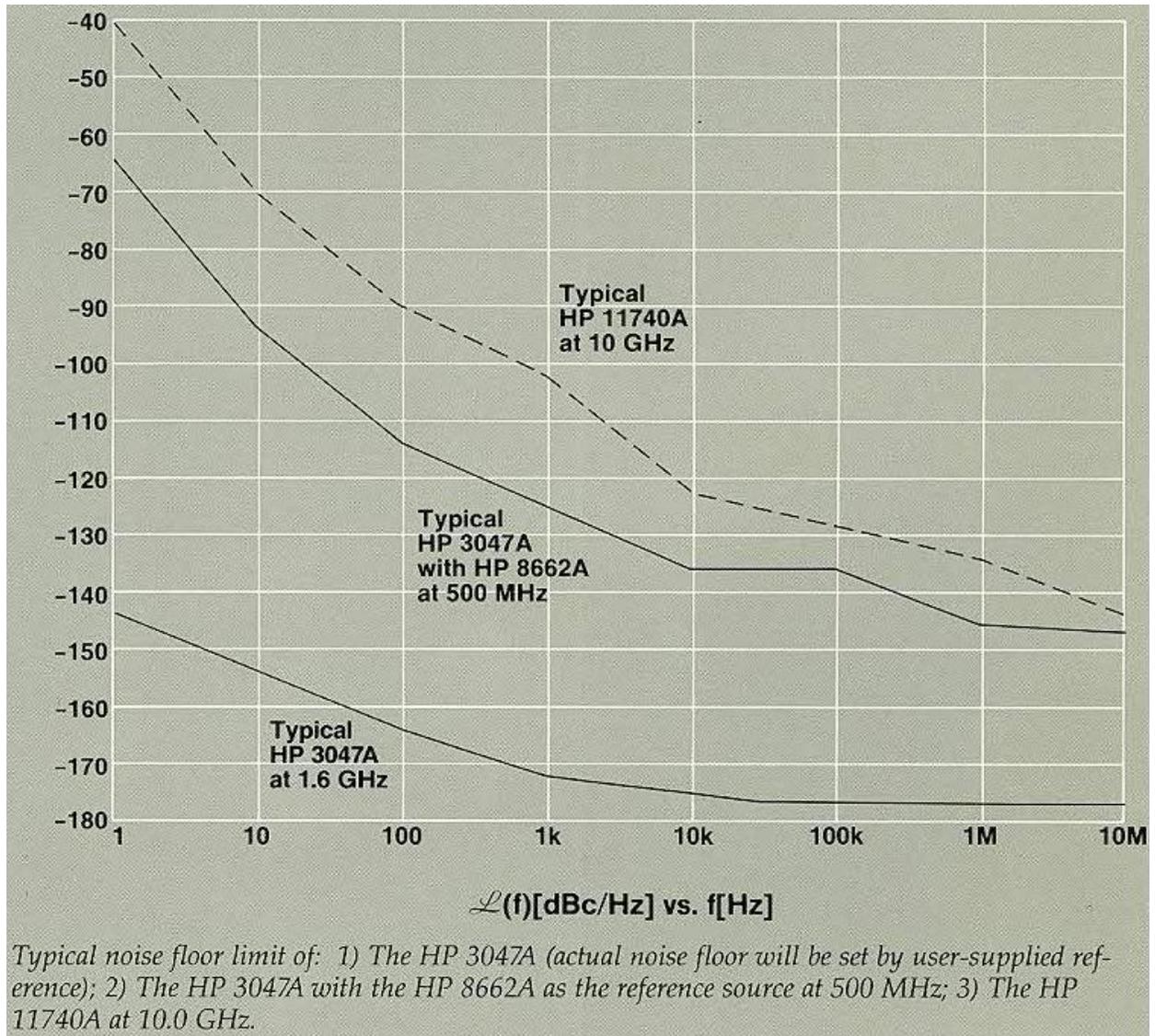
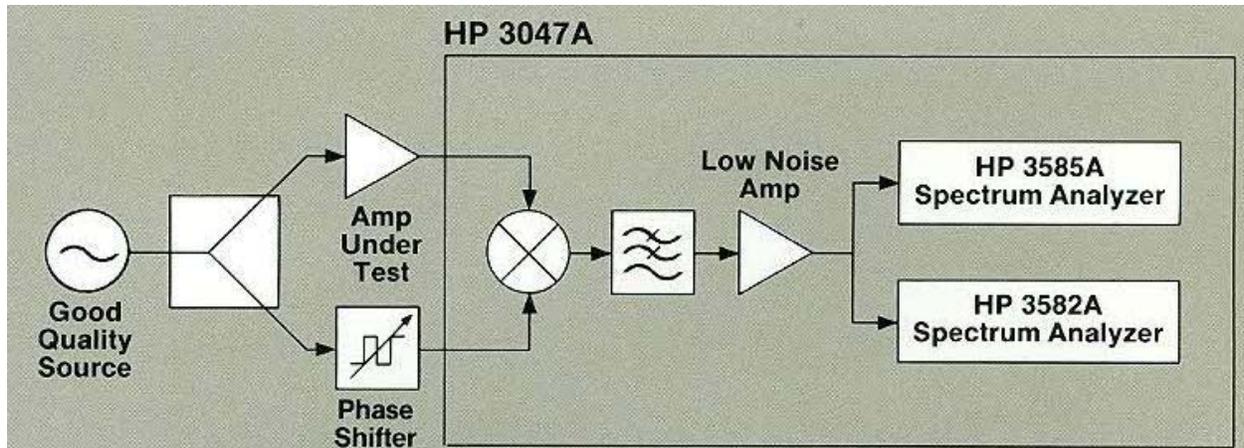


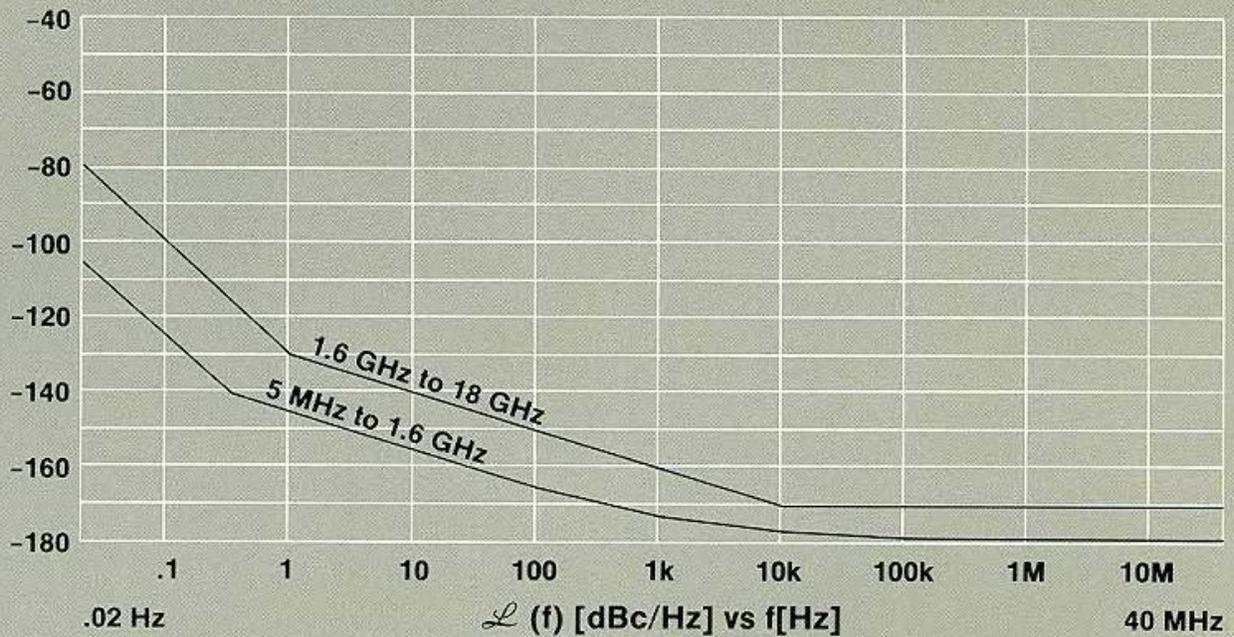
Figure 4: Typical Noise Floor limit

From a system's point-of-view, the numbers are not that optimistic, as seen from **Figure 5**. The reason for this is that the transistor used in the oscillator has its own noise contribution.

This system is used to measure residual noise in amplifiers or switches. This is an important diagram for system planning.



Measuring two-port phase noise on an amplifier with the HP 3047A.



HP 3047A system sensitivity for two-port phase noise measurements.

Figure 5: The HP3047A – Residual Phase Noise Measurement, not Oscillator Phase Noise

The HP3047A has a built-in crystal oscillator and its measured phase-noise is shown in Figure 6. The measurements are actually a little better than what was specified by HP but these are not as good as modern measurement systems.

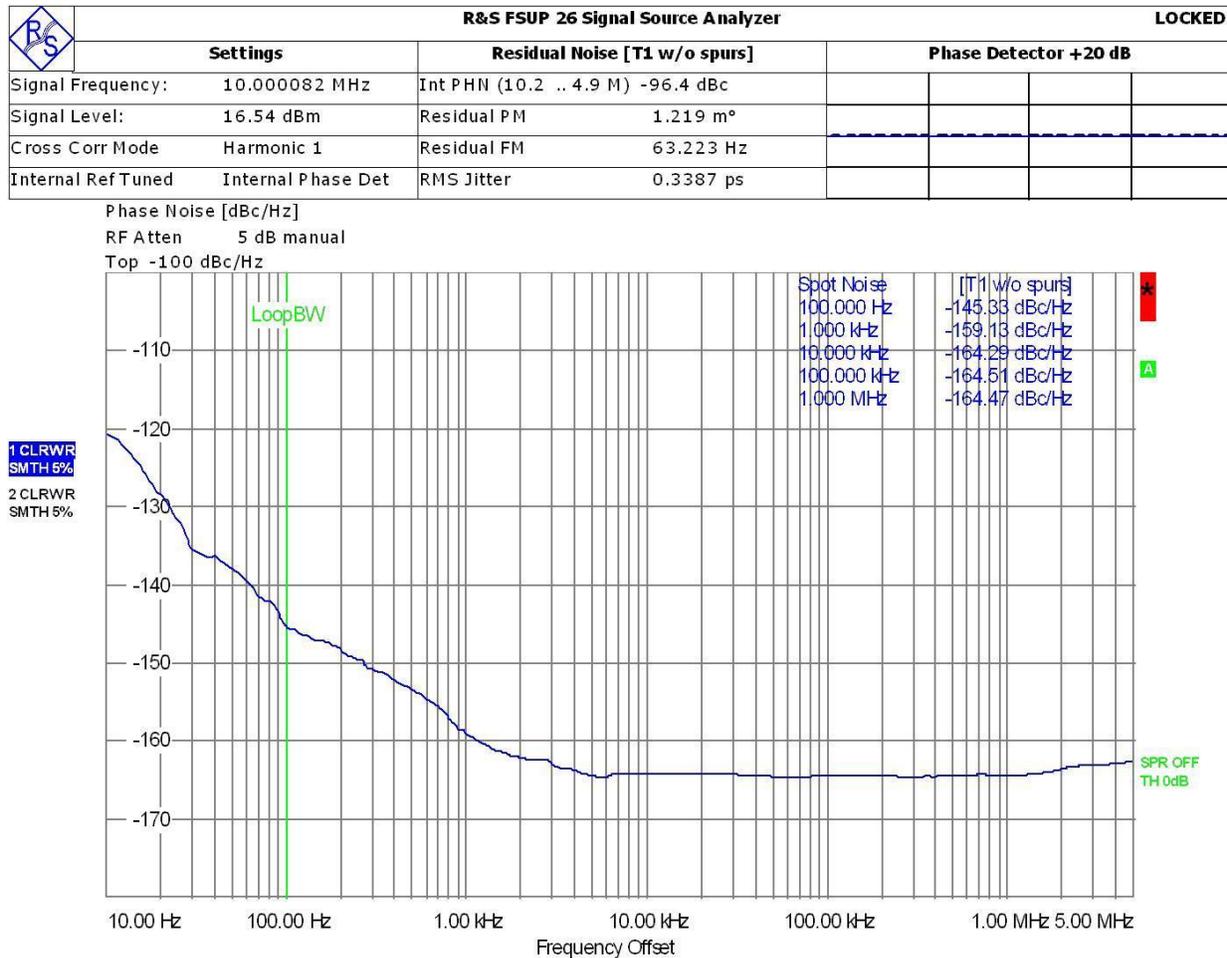


Figure 6: HP10811B measurement of 10MHz signal on R&S FSUP

Synergy kept the old HP system hardware for these references. The system is software based and used an FFT analyzer; for its time this was the best system around, but the measurements took a long time to make. The modern test equipments using the cross-correlation methods are at least 20 times faster.

Noise in Circuits and Semiconductors

Microwave applications generally use bipolar transistors and following are their noise contributions. ^{3, 5, 6, 9, 21}

Johnson noise

- The Johnson noise (thermal noise) is due to the movement of molecules in solid devices called Brown's molecular movements.
- This noise voltage is expressed as $v_n^2 = 4kT_0RB (emf) (volt^2)$

- The power of thermal noise can thus be written as

$$\text{Noise Power} = \frac{v_n^2}{4R} = kT_0B \text{ (W)}$$

It is most common to do noise evaluations using a noise power density, in Watts per Hz. We get this by setting B=1Hz. Then we get:

$$\text{For } B = 1\text{Hz, Noise Power} = kT_0$$

$$T = 290\text{K and } k - \text{ Boltzmann's constant} = 1.38 \times 10^{-23} \text{ J/K}$$

$$\text{by Thevinin, Noise Power} = 1.38 \times 10^{-23} \times 290 = 4 \times 10^{-23} \text{ W}$$

Noise floor below the carrier for zero dBm output is given by

$$L(\omega) = 10 \log \left(\frac{v_n^2/R}{1\text{mW}} \right) = -173.97\text{dBm or about } -174\text{dBm}$$

In order to reduce this noise, the only option is to lower the temperature, since noise power is directly proportional to temperature.

- The Johnson noise sets the theoretical noise floor.

Planck's Radiation Noise

- The available noise power does not depend on the value of resistor but it is a function of temperature T. The noise temperature can thus be used as a quantity to describe the noise behavior of a general lossy one-port network.
- For high frequencies and/or low temperature a quantum mechanical correction factor has to be incorporated for the validation of equation. This correction term results from Planck's radiation law, which applies to blackbody radiation.

$$P_{av} = kT \cdot \Delta f$$

$$P_{av} = kT \cdot \Delta f \cdot p(f, T), \text{ with } p(f, T) = \left[\frac{hf}{kT} / \left(e^{\left(\frac{hf}{kT} \right)} - 1 \right) \right]$$

$$\text{where } h = 6.626 \cdot 10^{-34} \text{ J} \cdot \text{s, Planck's Constant}$$

Schottky/Shot noise

- The Schottky noise occurs in conducting PN junctions (semiconductor devices) where electrons are freely moving. The root mean square (RMS) noise current is given by

$$\overline{i_n^2} = 2 \times q \times I_{dc}; \quad P = i_n^2 \times Z$$

Where, q is the charge of the electron, P is the noise power, and I_{dc} is the dc bias current.

Z is the termination load (can be complex).

- Since the origin of this noise generated is totally different, hence they are independent of each other.

Flicker noise

- The electrical properties of surfaces or boundary layers are influenced energetically by states, which are subject to statistical fluctuations and therefore, lead to the flicker noise or $1/f$ noise for the current flow.
- $1/f$ noise is observable at low frequencies and generally decreases with increasing frequency f according to the $1/f$ -law until it will be covered by frequency independent mechanism, like thermal noise or shot noise.

Example: The noise for a conducting diode is bias dependent and is expressed in terms of AF and KF.

$$\langle i_{Dn}^2 \rangle_{AC} = 2qI_{dc}B + KF \frac{I_{DC}^{AF}}{f} B$$

- The AF is generally in range of 1 to 3 (dimensionless quantity) and is a bias dependent curve fitting term, typically 2.
- The KF value is ranging from $1E^{-12}$ to $1E^{-6}$, and defines the flicker corner frequency.

Transit time and Recombination Noise

- When the transit time of the carriers crossing the potential barrier is comparable to the periodic signal, some carriers diffuse back and this causes noise. This is really seen in the collector area of NPN transistor.
- The electron and hole movements are responsible for this noise. The physics for this noise has not been fully established.

Avalanche Noise

- When a reverse bias is applied to semiconductor junction, the normally small depletion region expands rapidly.
- The free holes and electrons then collide with the atoms in depletion region, thus ionizing them and produce spiked current called the avalanche current.
- The spectral density of avalanche noise is mostly flat. At higher frequencies the junction capacitor with lead inductance acts as a low-pass filter.
- Zener diodes are used as voltage reference sources and the avalanche noise needs to be reduced by big bypass capacitors.

Prediction of Phase Noise

In designing an oscillator one needs to have a concept of how to do this and can hopefully validate the same. The basic equation (Rohde's Modified Leeson's Equation) needed to calculate the phase noise is found in *The Design of Modern Microwave Oscillators for Wireless Applications: Theory and Optimization*.^{1, 13, 14, 15, 16}

$$\mathcal{L}(f_m) = 10 \log \left\{ \left[1 + \frac{f_0^2}{[2f_m Q_0 m(1-m)]^2} \right] \left(1 + \frac{f_c}{f_m} \right) \frac{FkT}{2P_0} + \frac{2kTRK_0^2}{f_m^2} \right\}$$

where $\mathcal{L}(f_m)$, f_m , f_0 , f_c , Q_0 , F , k , T , P_0 , R , K_0 and m are the ratio of the sideband power in a 1Hz bandwidth at f_m to total power in dB, offset frequency, flicker corner frequency, loaded Q , unloaded Q , noise factor, Boltzmann's constant, temperature in degree Kelvin, average output power, equivalent noise resistance of tuning diode, voltage gain and ratio of the loaded and unloaded Q , respectively.

In the past this was done with the Leeson formula, which contains several estimates, those being output power, flicker corner frequency and the operating (or loaded) Q . Now, one can assume that the actual phase noise is never better than the **large signal noise figure (F)** given by the following equation. ^{13, 14}

$$F = 1 + \frac{Y_{21}^+ C_2 C_c}{(C_1 + C_2) C_1} \left[r_b + \frac{1}{2r_e} \left(r_b + \frac{(C_1 + C_2) C_1}{Y_{21}^+ C_2 C_c} \right)^2 \left(\frac{1}{\beta^+} + \frac{f^2}{f_T^2} \right) + \frac{r_e}{2} \right]$$

Applying the Cross-Correlation

The old systems have an FFT analyzer for close-in calculations and are slower in speed. Most modern equipments use the crystal oscillator using Rohde's design technique. The reason why the cross-correlation method became popular is that most oscillators have an output between zero and 10dBm and what is even more important is that only one single source is required. The method with a delay line, in reality required a variable delay line to provide correct phase noise numbers as a function of offset (this is shown in reference 1, pg 148-153, Fig 7.25 and 7.26).

This technique combines two duplicate single-channel reference sources/PLL systems and performs cross-correlation operations between the outputs of each channel, as shown in **Figure 7.7, 8**. The DUT noises through each channel are coherent and are not affected by the cross-correlation, whereas, the internal noises generated by each channel are incoherent and are diminished by the cross-correlation operation at the rate of M . (M being the number of correlation).

This can be expressed as:

$$N_{\text{meas}} = N_{\text{DUT}} + (N_1 + N_2)/M^{1/2}.$$

where, N_{meas} is the total measured noise at the display; N_{DUT} the DUT noise; N_1 and N_2 the internal noise from channels 1 and 2, respectively; and M the number of correlations. The two-channel cross-correlation technique achieves superior measurement sensitivity without requiring exceptional performance of the hardware components.

However, the measurement speed suffers when increasing the number of correlations⁵. The built-in synthesizer limits the dynamic range and that is the reason why at lower frequencies crystal-oscillators are used.

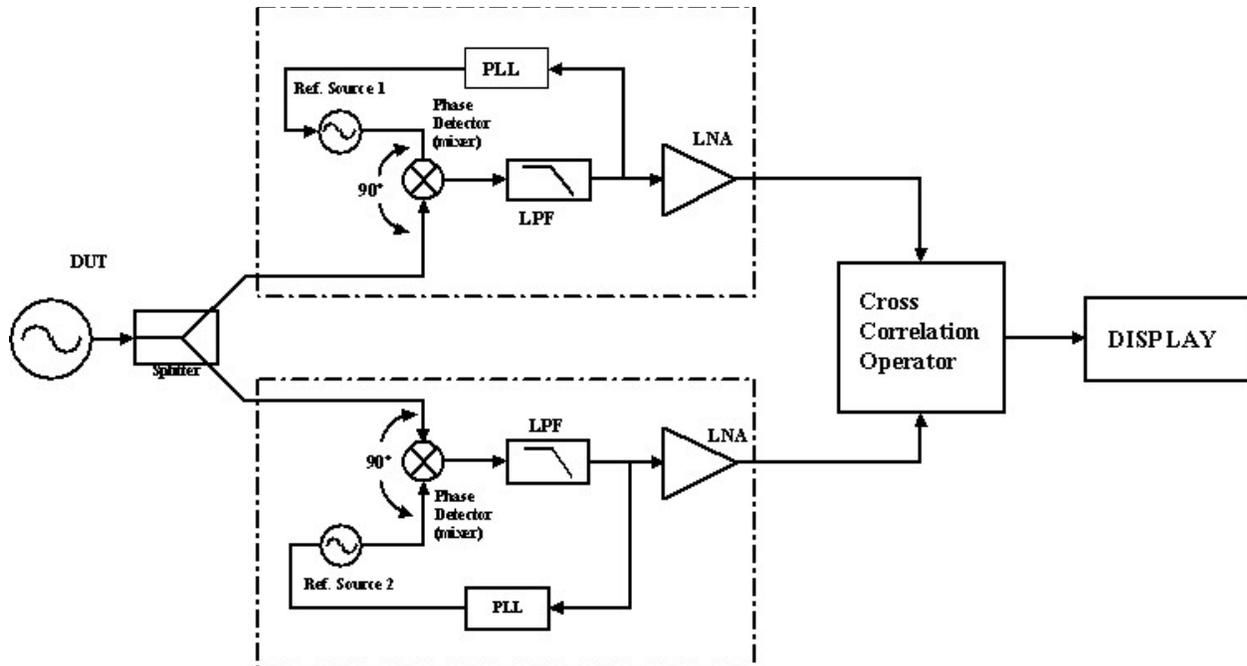


Figure 7: Block Diagram showing the Noise-Correlation Method

The assumption is that the internal reference source 1 and 2 are at least equal in noise contribution as the DUT and assuming a correlation of 20 dB (limited by leakage and other large signal phenomena) divider noise, those references can be 20 dB worse. Both R&S and Agilent use a synthesized approach. This limits the dynamic range close-in to the carrier and far-out to the carrier. When using the correlation to get 10 dB more dynamic range, 100 correlations are necessary or the measurement is one hundred times slower. The R&S FSUP is a combination of a spectrum analyzer and the phase noise test setup, while the Agilent E5052B is a dedicated system, about 4-6 times faster.

Advantages of the noise correlation technique: ¹¹⁻¹⁹

1. Increase speed
2. Requires less input power
3. Single source set-up
4. Can be extended from low frequencies like 1 MHz to 100 GHz

All of which depends upon the internal synthesizer

Disadvantages of the noise-correlation technique:

1. Different manufacturers have different isolation, so the available dynamic range is difficult to predict
2. These systems have a "sweet-spot", both R&S and Agilent start with an attenuator, not to overload the two channels; 1dB difference in input level can result in quite different measured numbers. These "sweet-spots" are different for each machine. The RF attenuation which is needed to find the "sweet-spot" reduces the overall dynamic range of the correlation by this amount.
3. The harmonic contents of the oscillator can cause an erroneous measurement [6], that's why a switchable-low-pass filter (such as the R&S Switchable VHF-UHF Low pass Filter Type PTU-BN49130 or its equivalent) should be used. ⁶

4. For frequencies below 200 MHz, systems such as Ana Pico or Holzworth using 2 crystal oscillators instead of a synthesizer must be used (there is no synthesizer good enough for this measurement). Example: Synergy LNX0100 Crystal Oscillator measures about -142dBc/Hz, (100 Hz offset), limited by the synthesizer of the FSUP and -147dBc/Hz with the Holzworth system. Agilent results are similar to the R&S FSUP, just faster.
5. At frequencies like 1 MHz off the carrier, these systems gave different results. The R&S FSUP, taking advantage of the “sweet-spot”, measures -183 dBc, Agilent indicates -175dBc/Hz and Holzworth measures -179 dBc/Hz.

We have not researched the “sweet-spots” for Agilent and Holzworth, but we have seen publications for both Agilent and Holzworth showing -190dBc/Hz far off the carrier. These were selected crystal oscillators from either Wenzel or Pascall.

Another problem is the physical length of the crystal oscillator connection cable to the measurement system. If the length provides something like “quarter-wave-resonance”, incorrect measurements are possible. The list of disadvantages is quite long and there is a certain ambiguity whether or not to trust these measurements or can they be repeated.

Here are some important examples of crystal oscillators. The first one is by Rohde (1975) and is shown in **Figure 8a** with simulated phase noise shown in **8b**.^{4,9}

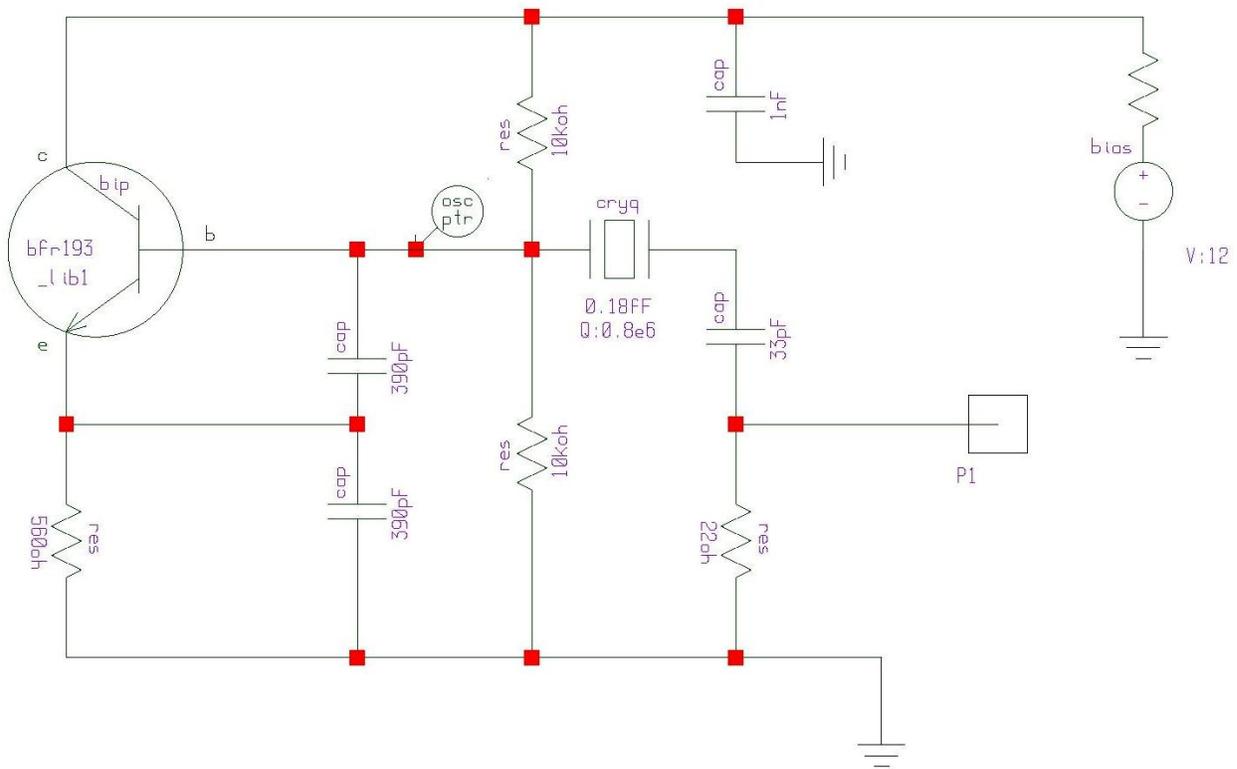


Figure 8a: 4 MHz Low Noise Crystal-Oscillator (U. L. Rohde, 1975)

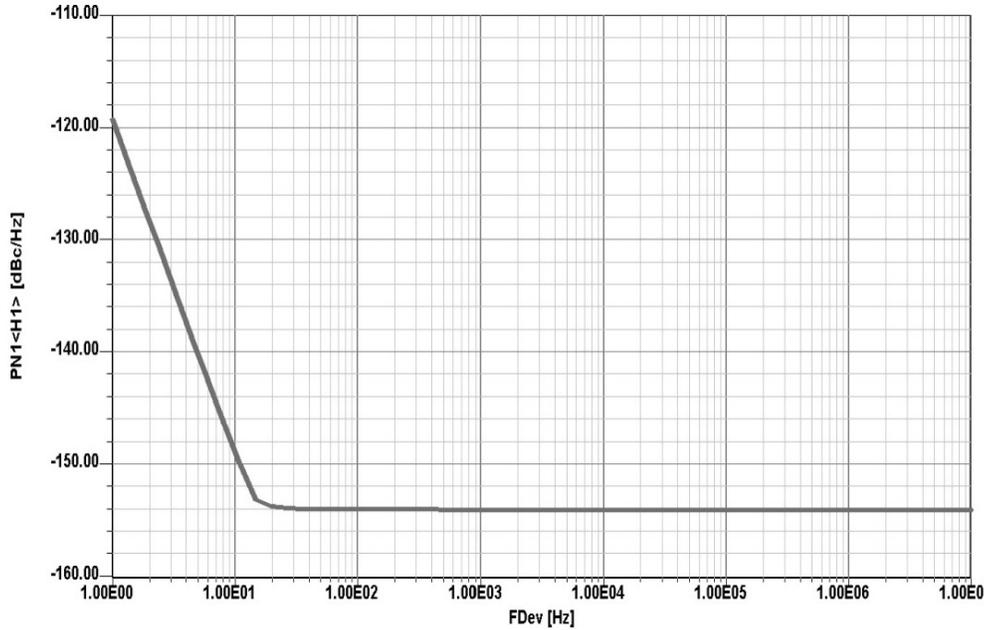


Figure 8b: Phase noise plot of the above circuit of Figure 8a

This represents the phase-noise of the oscillator without the buffer-amplifier shown in **Figure 8a**.

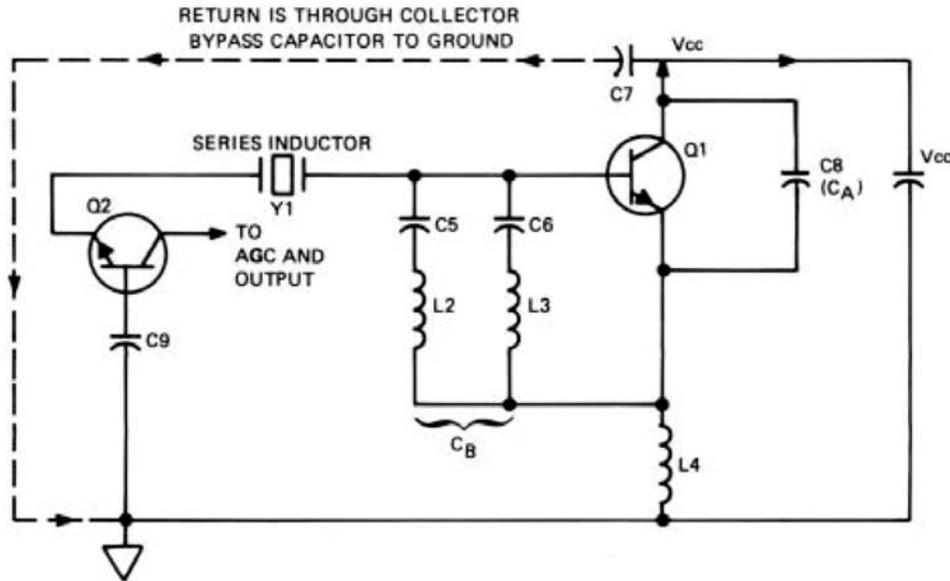


Figure 9: Rohde Design later implemented by HP

All modern equipment uses the crystal oscillator using Rohde's design technique (**Figure 9**). The actual measured numbers are shown in **Figure 6**. Another important oscillator is by Michael Driscoll and the circuit is shown in **Figure 10a** and its simulated phase noise in **10b**. The crystal is used as a filter to ground. We have seen applications of this design from 10 to 100MHz. Sometimes there is stability problem with upper transistor of the cascode. Another important contribution is

the introduction of the stress-compensated or doubly-rotated crystal, in short SC cut. This took over after the AT-cut. Its drawbacks are possible spurious resonance and higher cost, but the actual phase-noise is 6-10 dB less, and so far there is no clear explanation of why this is the case.^{10, 11}

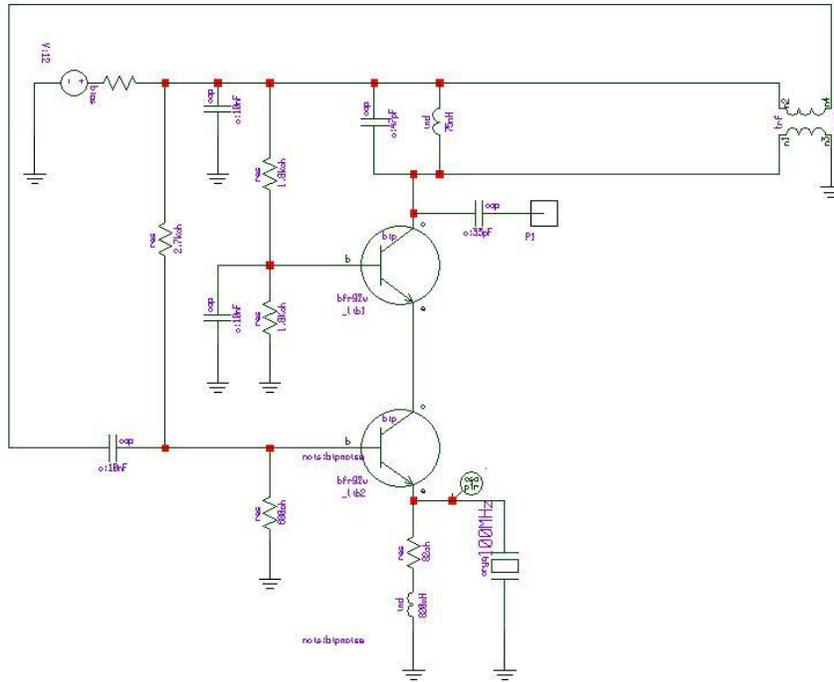


Figure 10a: Driscoll Oscillator

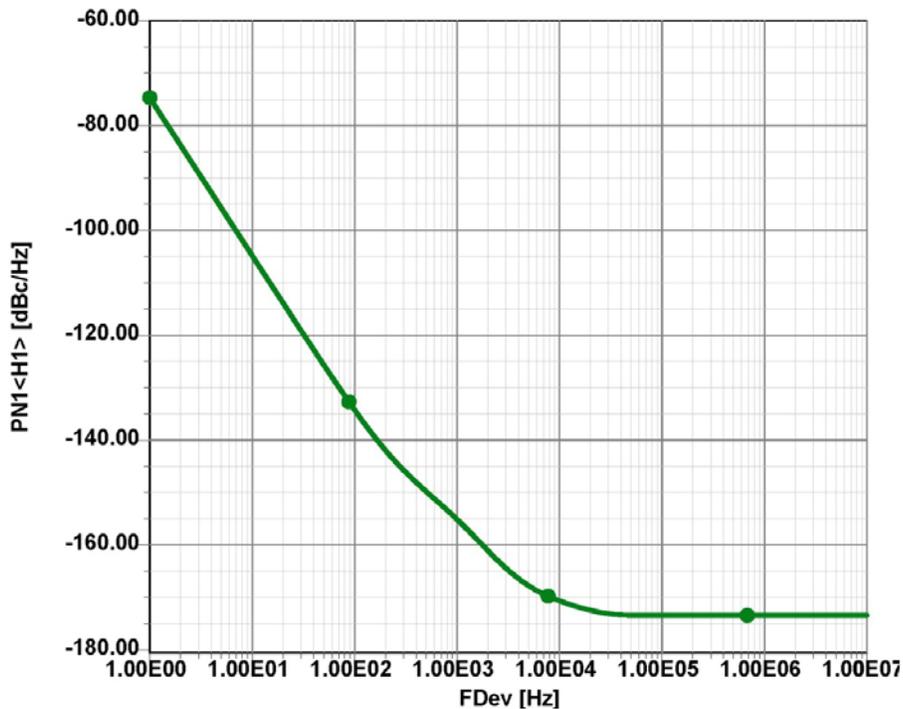


Figure 10b: Simulated Phase Noise plot of Driscoll Oscillator

The 100MHz crystal oscillator measurements are most demanding and these oscillators are the backbone of many test and communication equipment. Figure11 shows a typical simple 100 MHz Crystal Oscillator and its simulated phase noise. Its output power is shown in **Figure 11c**. This oscillator is missing a buffer stage.

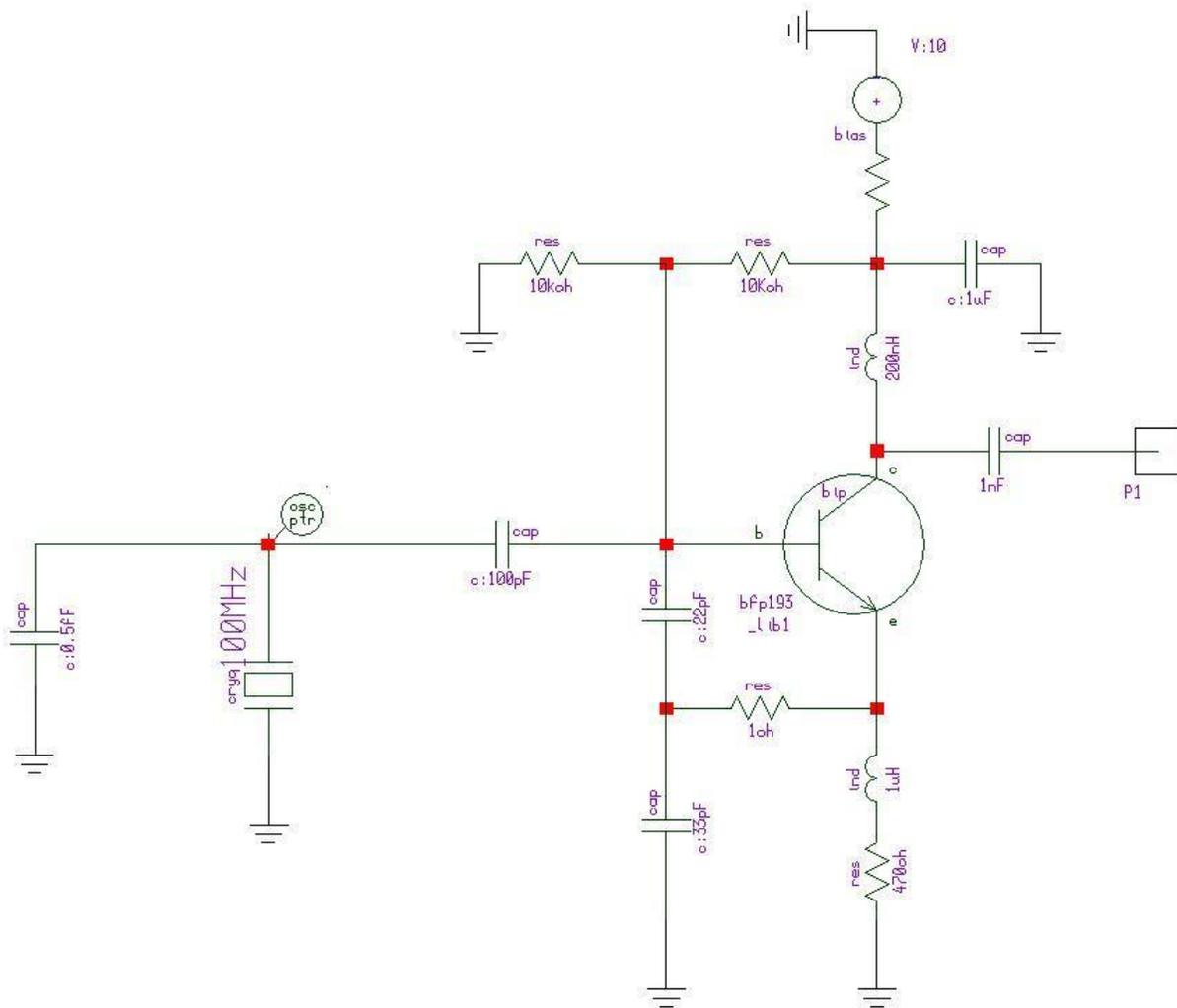


Figure 11a: 100MHz Crystal Oscillator without Buffer Stage

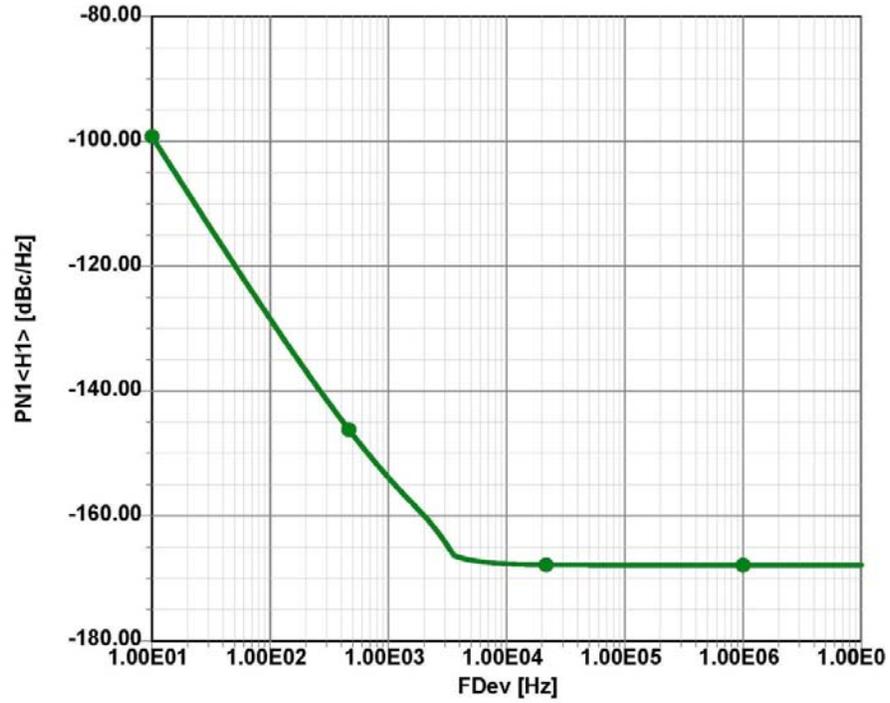


Figure 11b: Simulated Phase Noise Plot of 100MHz Crystal Oscillator without Buffer Stage

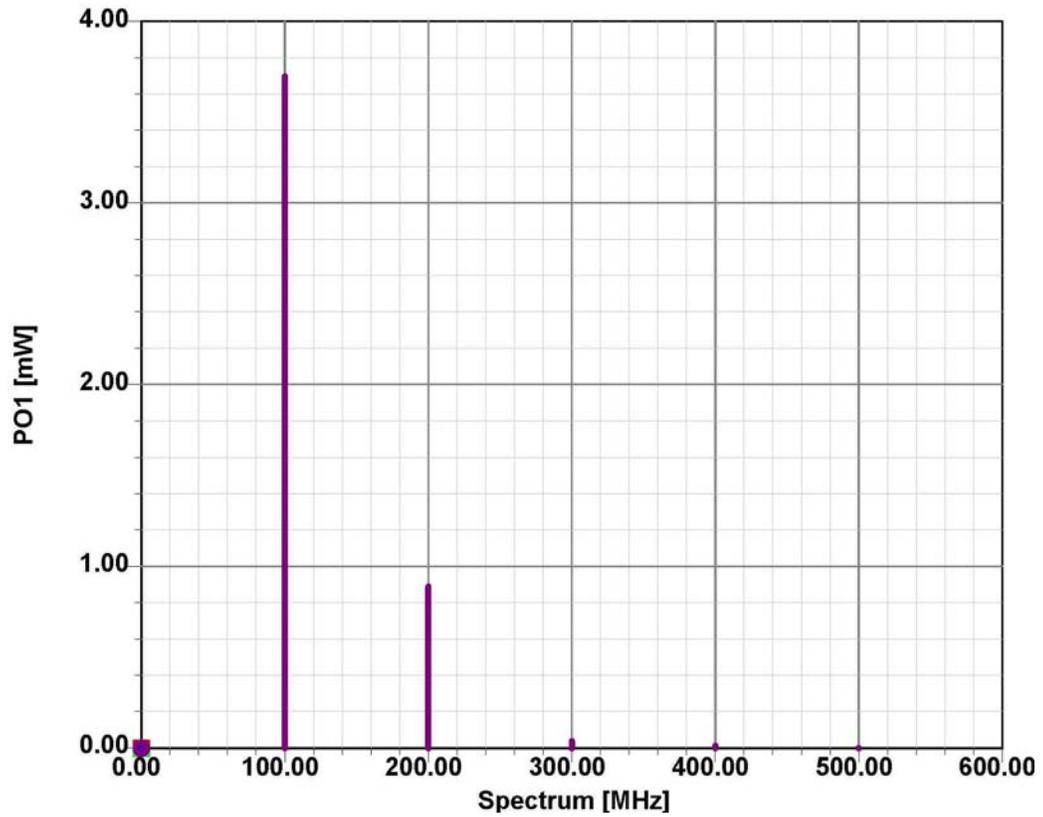


Figure 11c: Output Power of 100MHz Crystal Oscillator without Buffer Stage

Figure 12a shows the buffer amplifier Figure 12b its simulated phase noise and F_{min} .

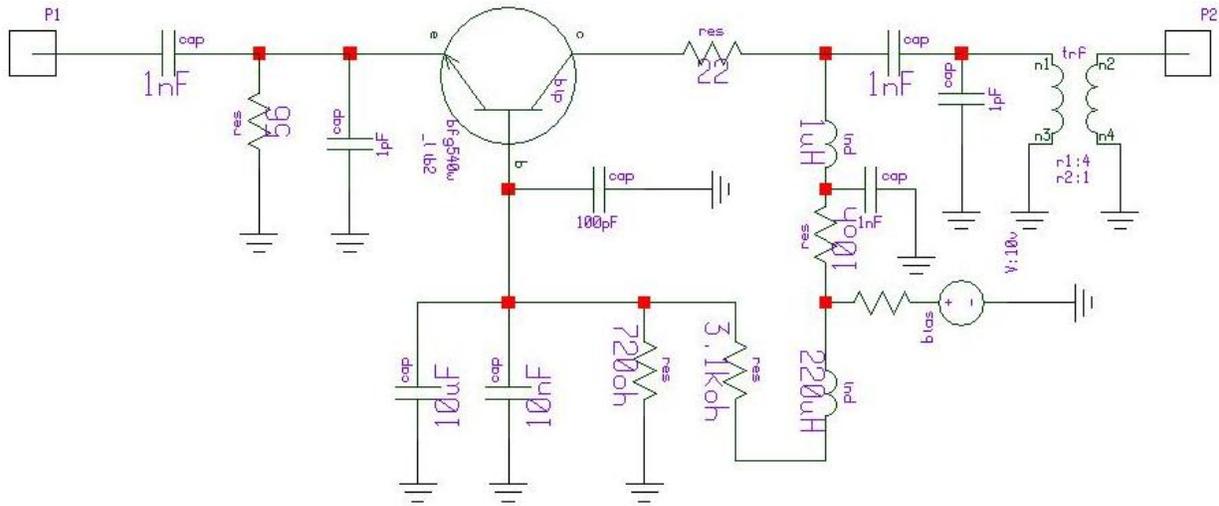


Figure 12a: Buffer Amplifier

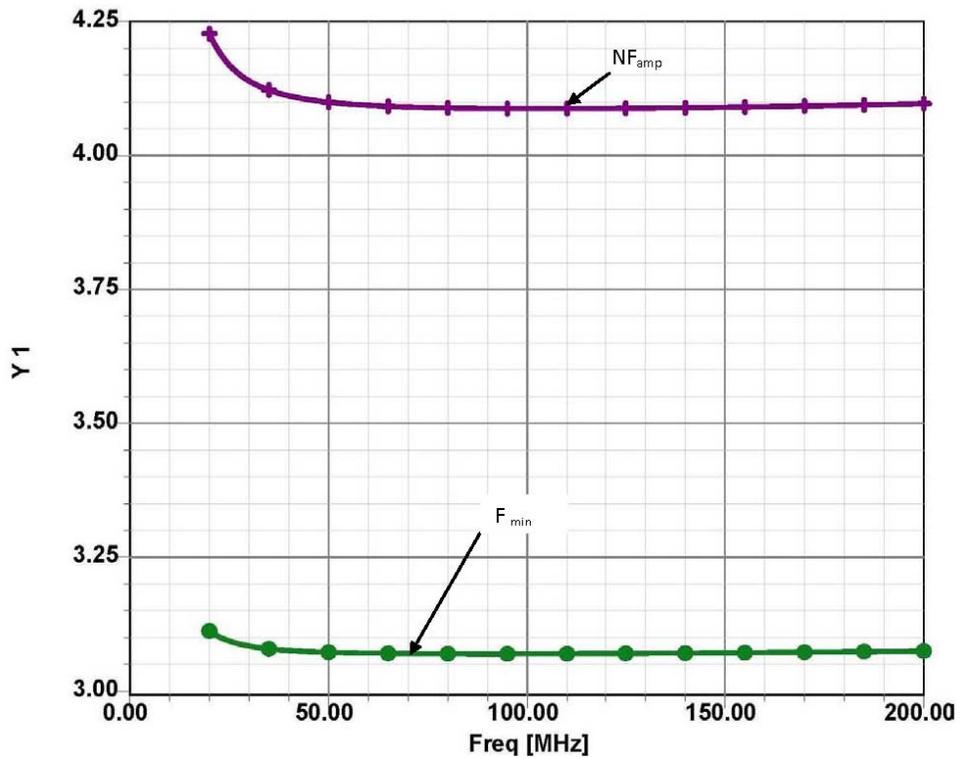


Figure 12b: Simulated Small Signal Noise Figure and F_{min} of the Buffer Amplifier shown in Fig 21

An increase in the noise figure, seen below 50MHz, is due to coupling capacitor.

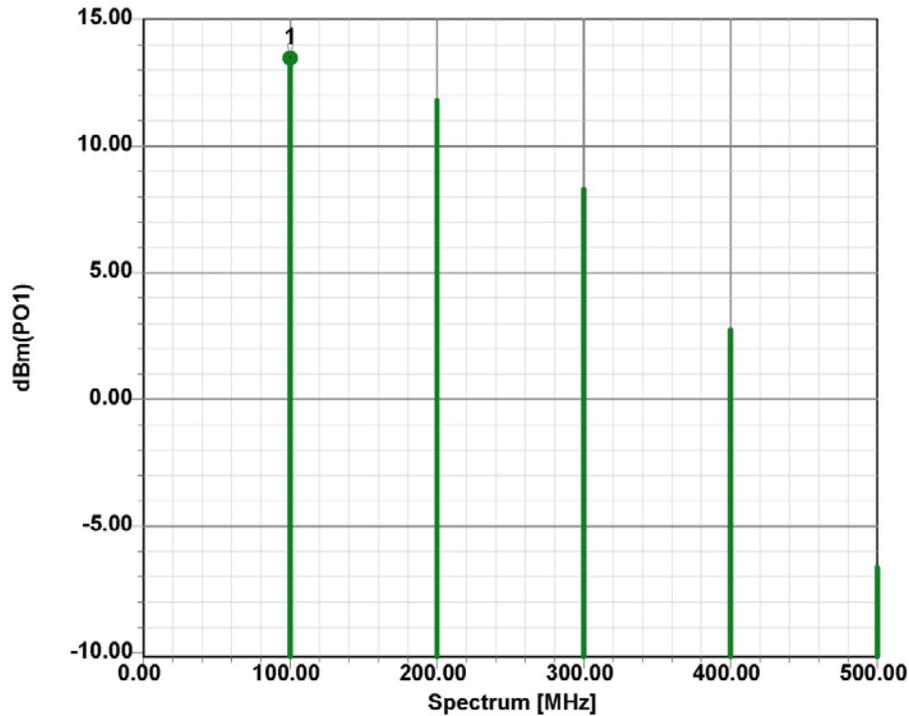


Figure 13c: Simulated Power Output Plot of 100MHz Crystal Oscillator with Buffer Stage

Figures 14 through 17 show measured results from the 100 MHz Crystal Oscillator performed on systems from Agilent, R&S, Holzworth and Ana Pico. **Appendix 1** shows the phase noise calculation for this oscillator. The 100 Hz offset phase noise is -146 dBc/Hz and the far out phase noise is -183 dBc/Hz.

These numbers agree well with the R&S FSUP measurements. The Agilent results are close, but we feel that the system is not optimized as the “sweet-spot” has not been characterized. This may be frequency and power level dependent. The Holzworth equipment shows the best close-in correct phase noise level, but we have not found the proper correlation figures. The Ana Pico test equipment seems to have a hump, but generally agrees well with the calculations. The results from the R&S FSUP, while taking a longer time (approximately 20 mins) are considered accurate from our experience. The Holzworth equipment performs the measurement in approximately 3-4 mins (5 correlations), and about 1 minute with no correlations and 3 dB worse phase noise.



Figure 26: 100MHz Crystal Oscillator Measured on Agilent E5052B (Corr_4000)

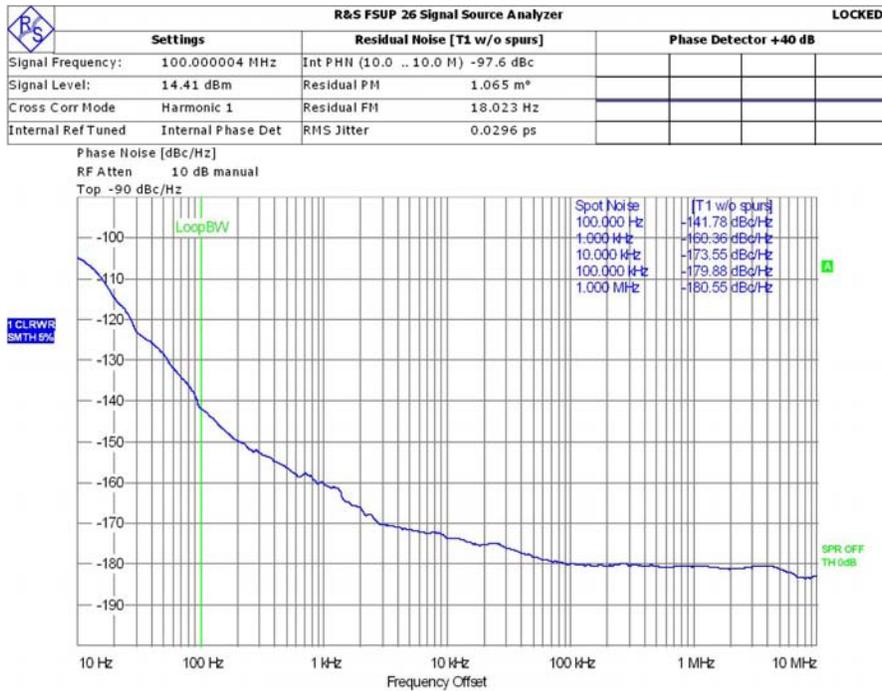


Figure 27: 100MHz Crystal Oscillator Measured on R&S FSUP

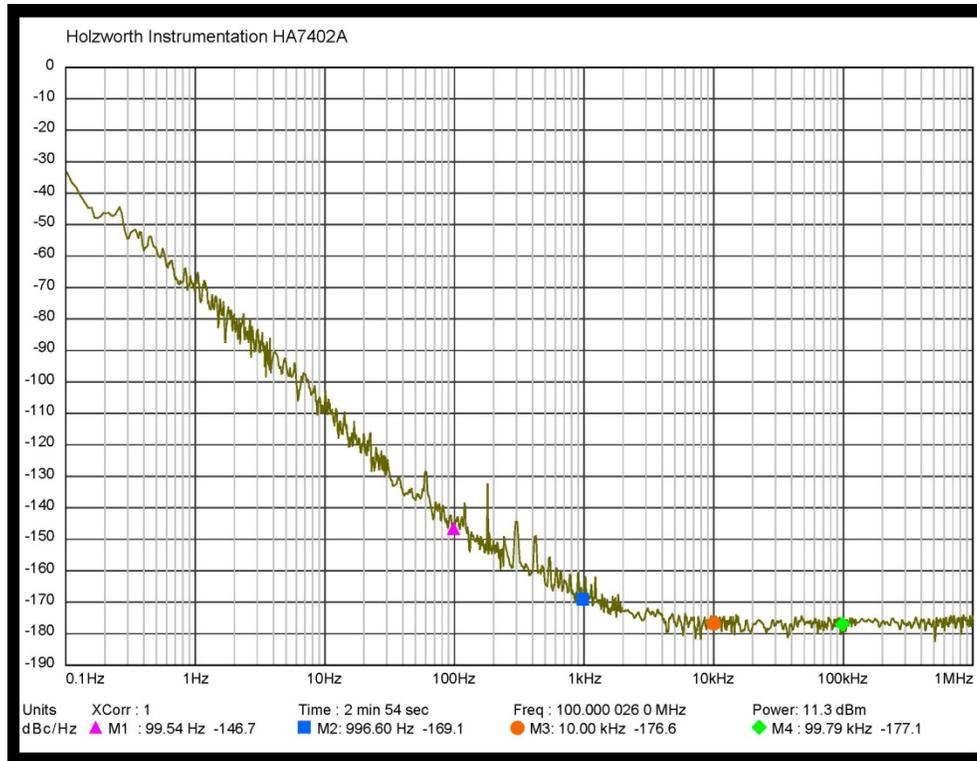


Figure 28: 100MHz Crystal Oscillator Measured on Holzworth Phase Noise Engine

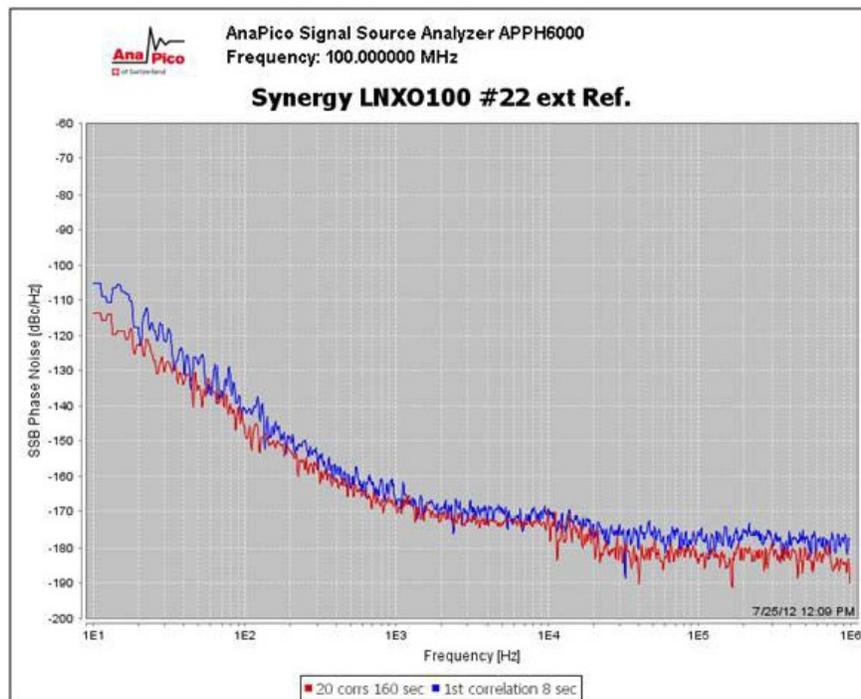


Figure 29: 100MHz Crystal Oscillator Measured on Anapico

What does the law of physics tell us? As pointed out above, here is the calculation for the Synergy LNXO100,

SSB phase noise = Pout (dBm) + 177 (dB) – NF (large signal noise figure of the buffer amplifier)

In our case, we get $14 + 177 - 7.7 = 183.3$.

This means the SSB phase noise far out is -183.3dBc/Hz.

Conclusion

In this paper, we have looked at typical oscillator circuits and some of their design rules. Calculations for phase noise were reviewed. Various systems were reviewed measuring a 100 MHz Crystal Oscillator. The phase noise equations give the best possible phase noise. If the equipment in use, after many correlations gives a better number, it violates the laws of physics as we understand them and if it gives a worse number, then either the correlations settings need to be corrected or the dynamic range of the equipment is insufficient. We realize that this treatment is exhaustive, but we think that it was necessary to explain how things fall in place. At 20 dBm output, the output amplifier certainly has a higher noise figure, as it is driven with more power and there is no improvement possible. There is an optimum condition and some of the measurements showing -190dBc/Hz do not seem to match the theoretical calculations. The correlation allows us to look below kT , but the noise contribution below kT is as useful as finding one gold atom in your body's blood. This gold atom has no contribution to your system.

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Appendix I

Phase noise Plot based on analytical formulae see [1]
 After defining all the values the phase noise can be correctly predicted

$C1 := 15.6 \text{ pF}$ $q_{\text{charge}} := 1.602 \cdot 10^{-19} \cdot \text{coul}$ $y11 := (0.000884 - 0.0000158j) \cdot \text{mho}$
 $C2 := 12.5 \text{ pF}$ $R := 0.2 \cdot \Omega$ $y21 := (-0.0105 - 0.00084j) \cdot \text{mho}$
 $V_{\text{cc}} := 10$ $L := 1600 \cdot 10^{-9} \cdot \text{henry}$ $I_{\text{c}} := 28 \cdot \text{mA}$
 $i := 0..7$ $p := 1.45$ $q := 1.05$ $I_{\text{b}} := 220 \cdot \mu\text{A}$ $Q_{\text{mac}} := 377$ $af := 2$ $Q := 200000$
 $f_{\text{c}} := 100 \cdot \text{MHz}$ $\omega_{\text{c}} := 2 \pi \cdot f_{\text{c}}$ $\text{nfdB} := 7.7$ $k_{\text{f}} := 1 \cdot 10^{-10}$
 $f_{\text{o}_i} := 10^i \cdot \text{Hz}$ $\omega_{\text{o}_i} := 2 \pi \cdot f_{\text{o}_i}$ $\text{PoutdB} := 14$ $KT := 4.143 \cdot 10^{-21} \cdot \text{J}$
 $y := \frac{C1}{C2}$ $B1_i := (\omega_{\text{o}_i})^2 \cdot L^2 \cdot V_{\text{cc}}^2$
 $k_{\text{constant}} := \frac{KT \cdot R}{\omega_{\text{c}}^2 \cdot C2^2}$ $b := \frac{|y21| \cdot y^p}{|y11|}$ $gm := |y21| \cdot y^q$ $k0_i := \frac{k_{\text{constant}}}{B1_i}$
 $k1_{\text{constant}_i} := q_{\text{charge}} \cdot I_{\text{c}} \cdot gm^2 + \frac{k_{\text{f}} \cdot I_{\text{b}}^{af} \cdot gm^2}{\omega_{\text{o}_i}}$ $t2_i := k0_i \cdot \frac{(1+y)^2}{y^2}$
 $k1_i := \frac{k1_{\text{constant}_i}}{\omega_{\text{c}}^2 \cdot B1_i}$ $k3_i := \omega_{\text{c}}^2 \cdot gm^2$ $k2_i := \omega_{\text{c}}^4 \cdot b^2$
 $k_i := \frac{k3_i}{k2_i \cdot C2^2}$ $t1_i := \left[\left(\frac{b^2}{gm^3} \right)^2 \cdot \frac{(k_i)^3 \cdot k1_i \cdot (\omega_{\text{c}})^2}{(y^2 + k_i)} \right] \cdot \frac{(1+y)^2}{y^2}$
 $l_i := t1_i + t2_i$
 $m_i := 10 \cdot \log \left[l_i \cdot \left(\text{kg}^{-2} \cdot \text{m}^{-4} \cdot \text{s}^5 \cdot \text{A}^2 \right) \cdot \frac{Q_{\text{mac}}^2}{Q^2} \right]$
 $L_i := \text{if} [m_i < (-(177 + \text{PoutdB} - \text{nfdB})), (-(177 + \text{PoutdB} - \text{nfdB})), m_i]$

m_i
- 88.035
- 117.814
- 146.064
- 169.714
- 190.327
- 210.394
- 230.4
- 250.401

f_{o_i}
$1 \cdot \text{s}^{-1}$
$10 \cdot \text{s}^{-1}$
$100 \cdot \text{s}^{-1}$
$1 \cdot 10^3 \cdot \text{s}^{-1}$
$1 \cdot 10^4 \cdot \text{s}^{-1}$
$1 \cdot 10^5 \cdot \text{s}^{-1}$
$1 \cdot 10^6 \cdot \text{s}^{-1}$
$1 \cdot 10^7 \cdot \text{s}^{-1}$

L_i
- 88.035
- 117.814
- 146.064
- 169.714
- 183.3
- 183.3
- 183.3
- 183.3

Figure I: Calculations for Phase Noise for 100MHz Crystal Oscillator

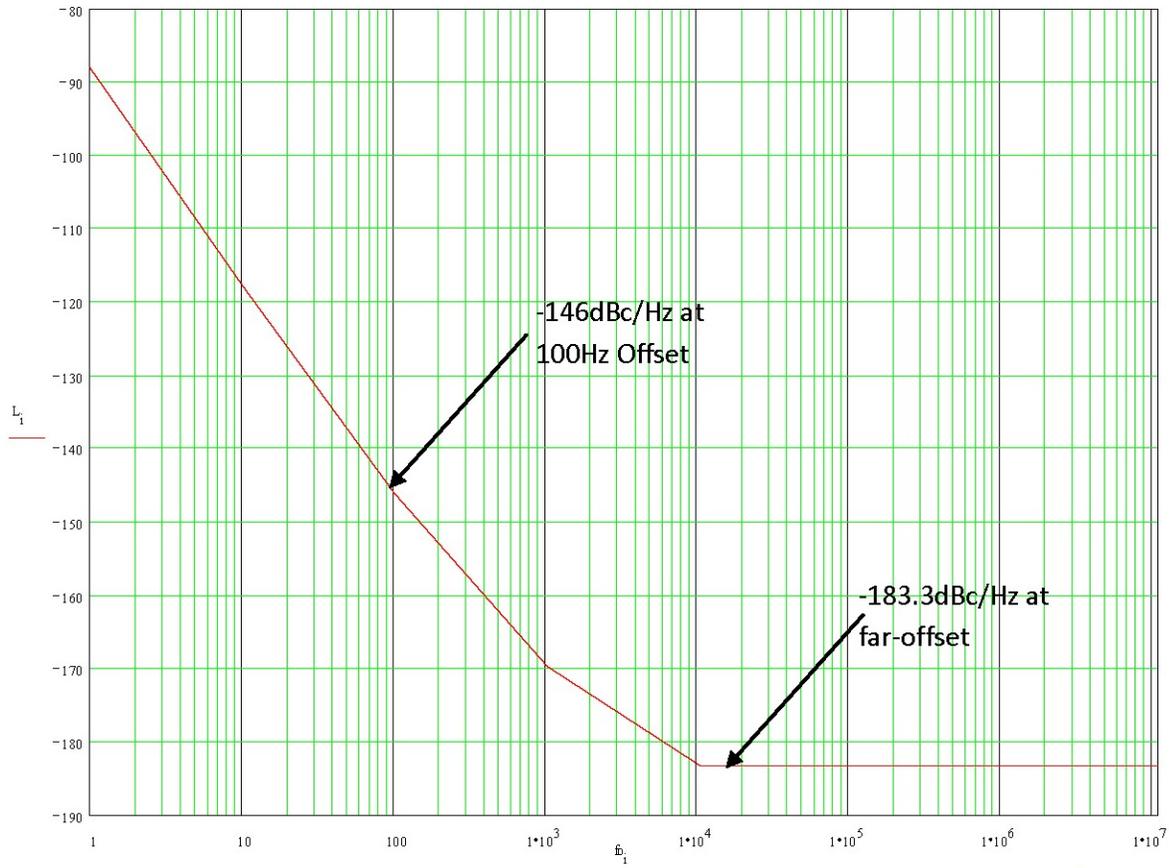


Figure II: Calculated Phase Noise plot for 100MHz Crystal Oscillator